Research
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ABSTRACTS: AMSI-ANZIAM LECTURER 2011 - PROFESSOR DARREN CROWDY

Complex analysis in low Reynolds number hydrodynamics

It is a well-known fact that the methods of complex analysis provide great advantage in studying physical problems involving a harmonic field satisfying Laplace's equation. One example is in ideal fluid mechanics (infinite Reynolds number) where the absence of viscosity, and the assumption of zero vorticity, mean that it is possible to introduce a so-called complex potential – an analytic function from which all physical quantities of interest can be inferred. In the opposite limit of zero Reynolds number flows which are slow and viscous and the governing fields are not harmonic it is much less common to employ the methods of complex analysis even though they continue to be relevant in certain circumstances.

This talk will give an overview of a variety of problems involving slow viscous flows where complex analysis can be usefully employed to gain theoretical insights. A number of example problems will be considered including problems of viscous sintering and the manufacture of microstructured optic fibres, the locomotion of low-Reynolds-number micro-organisms and the friction prop- erties of superhydrophobic surfaces in microfluidics.

Solving problems in multiply connected domains

Motivated by problems arising in the applied sciences, this talk surveys a new theoretical approach to solving problems in multiply connected planar domains as developed by the speaker (and his group) in recent years. Multiply connected domains are "regions with holes" and are ubiquitous in applications; whenever two or more objects/entities interact in some ambient medium the analysis may call for the methods discussed in this lecture.

We will advocate the use of ideas from constructive function theory and complex analysis to provide quasi-analytical solutions to such problems in terms of the so-called "Schottky-Klein prime function" – a very important classical special function that is hardly known to nonspecialists but which is relevant to a surprisingly wide range of applied mathematical problems often facilitating concise and elegant representations of their solutions. While the theory of the prime function was first discussed well over a hundred years ago, it has hardly been used in applied problems and it is only recently that numerical methods have been developed to actually evaluate it in a robust manner.

Some illustrative example problems from applications (e.g. fluid dynamics, potential theory, conformal mapping) will be described and their solutions explicitly constructed. We will also describe freely available numerical codes that we have developed for the computation of the prime function in order to promote its use.

We hope to demonstrate that the new methods are sufficiently general that they provide broad scope for tackling a variety of mathematical problems.

A new calculus for ideal fluid dynamics

In classical fluid dynamics, an important basic problem is to understand how solid bodies (e.g. aerofoils, obstacles or stirrers) immersed in an ideal fluid interact by "communicating" with each other through the ambient fluid. Also of interest is how vortices interact with such solid bodies (and each other). There is great interest in such problems in areas such as aerodynamics, biolocomotion and oceanography.

For two-dimensional flows, a variety of powerful mathematical results exist (complex variable methods, conformal mapping, Kirchhoff-Routh theory) that have been used to study such problems, but the constructions are usu- ally restricted to problems with just one, or perhaps two, objects. Expressed mathematically, most studies deal only with fluid regions that are simply or doubly connected. There has been a general and longstanding perception that problems involving fluid regions of higher connectivity – that is, more than two interacting objects – are too intractable to be tackled analytically (and that numerical methods must be used).

The lecture will show that there is a way to formulate the theory so that the relevant fluid dynamical formulae are exactly the same irrespective of the number of interacting objects (i.e., the approach is relevant to fluid domains of any finite connectivity). This provides a flexible and unified tool (a "calculus") for modelling the fluid dynamical interaction of multiple objects/aerofoils/obstacles in ideal flow as well as their interaction with free vortices. Examples of how to apply the calculus to specific problems will be given to illustrate its flexibility.

More generally, the results have wider reaching applications beyond fluid dynamics and essentially provide a new calculus for two-dimensional potential theory.

A complex life

"But think of Adam and Eve like an imaginary number, like the square root of minus one: you can never see any concrete proof that it exists, but if you include it in your equations, you can calculate all manner of things that couldn't be imagined without it."

The Golden Compass, Philip Pullman

Imaginary, or complex, numbers have long fascinated not just mathematicians but the public at large; it is bemusing and intriguing that an "imagined" abstraction can have real-life utility. At a time when the teaching of complex analysis to undergraduate engineers, and even to undergraduate mathematicians, is arguably in decline I will present evidence, drawn mainly from my own research interests, that complex analysis continues to be an indispensable mathematical tool with perennial relevance to modern day applications in science and engineering.