AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE | AMSI Online | HONOURS & MASTERS
EXPANDING ACCESS TO ADVANCED MATHEMATICAL SCIENCES

AMSI Online: Honours and Masters Subject Guide

Elliptic Functions, Elliptic Curves and Modular Forms

Semester One, 2026

Administration and contact details

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Subject details

Handbook entry URL	
Subject homepage URL	
Honours student hand-out URL	
Teaching period (start and end date):	**16 February – 24 April 2025**
Exam period (start and end date):	**27 April – 7 May 2025**
Contact hours per week:	**3**
ACE enrolment closure date:	**16 February 2025**
Lecture day(s) and time(s):	**TBC**
Description of electronic access arrangements for students (for example, LMS)	**Canvas LMS**

Subject content

1. **Subject content description**

   We all know that trigonometric functions are periodic, e.g. $\cos(x + 2\pi) = \cos(x)$, so cos has period 2π. *Elliptic functions* are complex functions which are doubly periodic, i.e. they have two linearly independent periods. Elliptic functions were discovered in attempts to compute the circumference of an ellipse (hence the name) but have taken up a life of their own. Just like cos and sin parametrise the circle $x^2 + y^2 = 1$, so elliptic functions parametrise *elliptic curves* defined by $y^2 = 4x^3 + g_2 x + g_3$.

   Elliptic curves, meanwhile, play a central role in number theory and cryptography. Varying elliptic curves (by varying the periods of the underlying elliptic functions) leads to *modular forms*, which also play a central role in number theory – Marting Eichler once quipped that "the five basic operations of arithmetic are addition, subtraction, multiplication, division and modular forms."

   In this course, we will start with elliptic functions, then move to elliptic curves (over the complex numbers and also other fields) and outline how they fit into various modern topics. Finally, we will study modular forms and see how they allow us to solve various problems from number theory.

2. **Week-by-week topic overview**

3. **Assumed prerequisite knowledge and capabilities**

   A first course in complex analysis is all that is required.

   Familiarity with some abstract algebra and number theory would be helpful.

4. **Learning outcomes and objectives**

   Students will

   - Understand basic concepts of elliptic functions, elliptic curves and modular forms
   - Appreciate the interrelations between these topics and the rest of Mathematics
   - Solve problems and write their solutions as clear, correct mathematical arguments.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Insert Program Learning Outcome here	Choose from list below

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Course notes will be provided as the course progresses. Useful reference include:

- J.H. Silverman, "The arithmetic of elliptic curves" (Chapter VI)
- J.H. Silverman, "Advanced topics in the arithmetic of elliptic curves" (Chapter I)
- M. Köcher & A. Krieg, "Elliptische Functionen und Modulformen" (in German, sorry)
- T. Tao, "246B, Notes 3: Elliptic functions and modular forms" (https://terrytao.wordpress.com/2021/02/02/246b-notes-3-elliptic-functions-and-modular-forms/)

6. Assessment breakdown

Exam	**50%**
Assignment	**50%**
Class work	Enter %

Assignment due dates	**Exam date (approximate)**
6 March, 17 April 2025	**4 May 2025**
Click here to enter a date.	
Click here to enter a date.	
Click here to enter a date.	

Institution honours program details

Weight of subject in total honours assessment at host department	**10 units of 80 total**
Thesis/subject split at host department	**40 units of 80 total**
Honours grade ranges at host department	
H1	85-100
H2a	75-84
H2b	65-74
H3	50-64

Institution masters program details

Weight of subject in total masters assessment at host department	
Thesis/subject split at host department	
Masters grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %