



Semester 1 Diagnostic Quiz, 2026

School of Mathematics and Statistics

## **MAST90103 Random Matrix Theory**

Submission deadline: None

This assignment consists of 9 pages (including this page)

**Question 1**

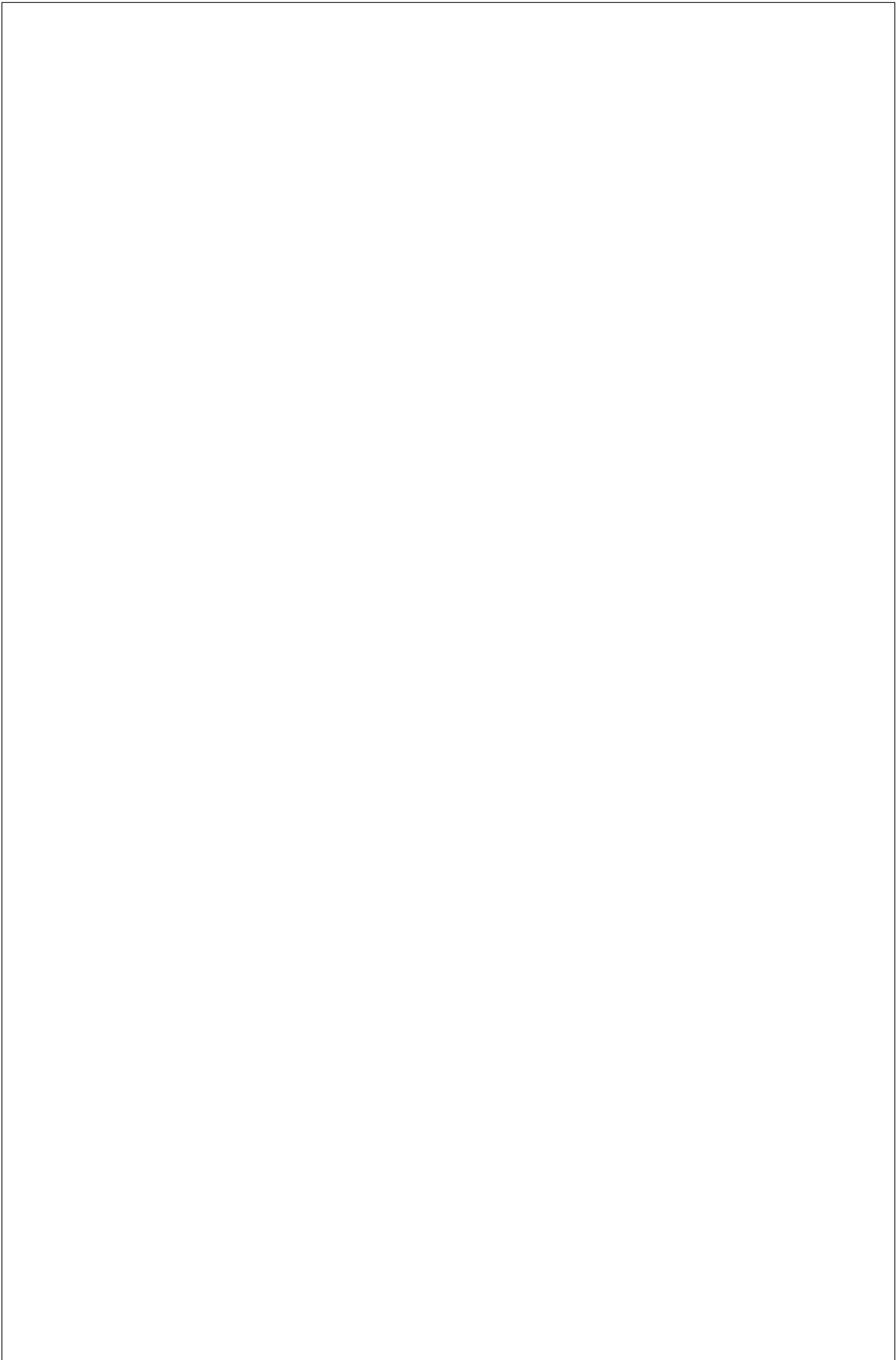
Consider five matrices  $A, B \in \mathbb{C}^{N \times N}$ ,  $C \in \mathbb{C}^{N \times M}$ ,  $D \in \mathbb{C}^{M \times N}$ , and  $E \in \mathbb{C}^{M \times M}$  of dimensions  $N \times N$ ,  $N \times M$ ,  $M \times N$  and  $M \times M$ , respectively. Additionally, the matrix  $E$  should be invertible. Show the following three identities for the determinant.

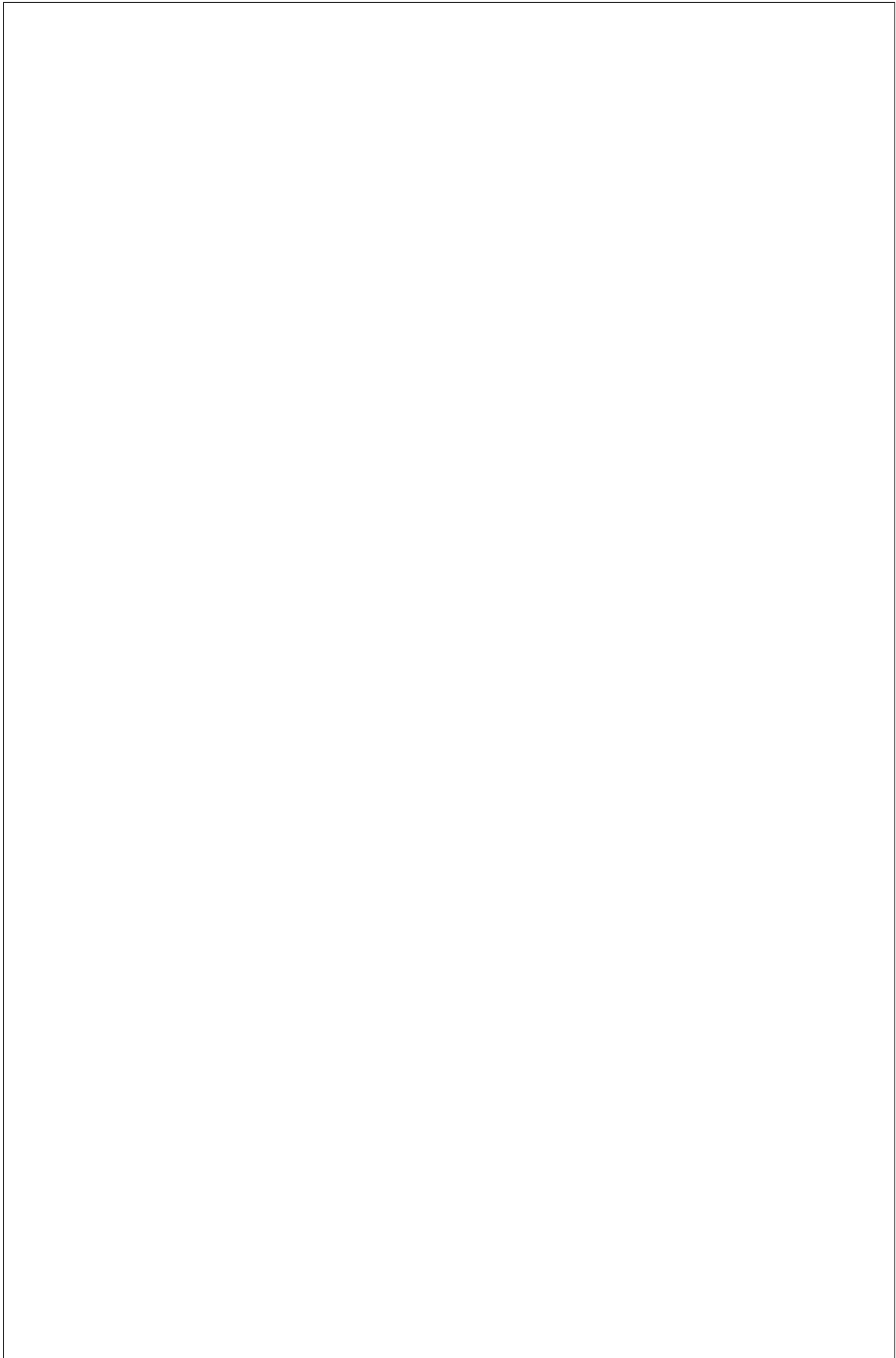
- (a)  $\det[AB] = \det[A] \det[B]$ , prove this by the Leibniz formula of the determinant

$$\det[A] = \sum_{\omega \in \mathbb{S}_N} \text{sign}(\omega) \prod_{j=1}^N A_{j\sigma(j)}$$

with  $\mathbb{S}_N$  the symmetric group comprising all permutations of  $N$  elements and  $\text{sign}(\omega)$  is  $+1$  when  $\omega$  is an even permutation and  $-1$  when it is an odd one;

- (b)  $\det \begin{bmatrix} A & C \\ D & E \end{bmatrix} = \det[A - CE^{-1}D] \det[E]$ , prove this by using identity (a);
- (c)  $\det[\mathbf{1}_N - CD] = \det[\mathbf{1}_M - DC]$ , prove this by using identity (b).

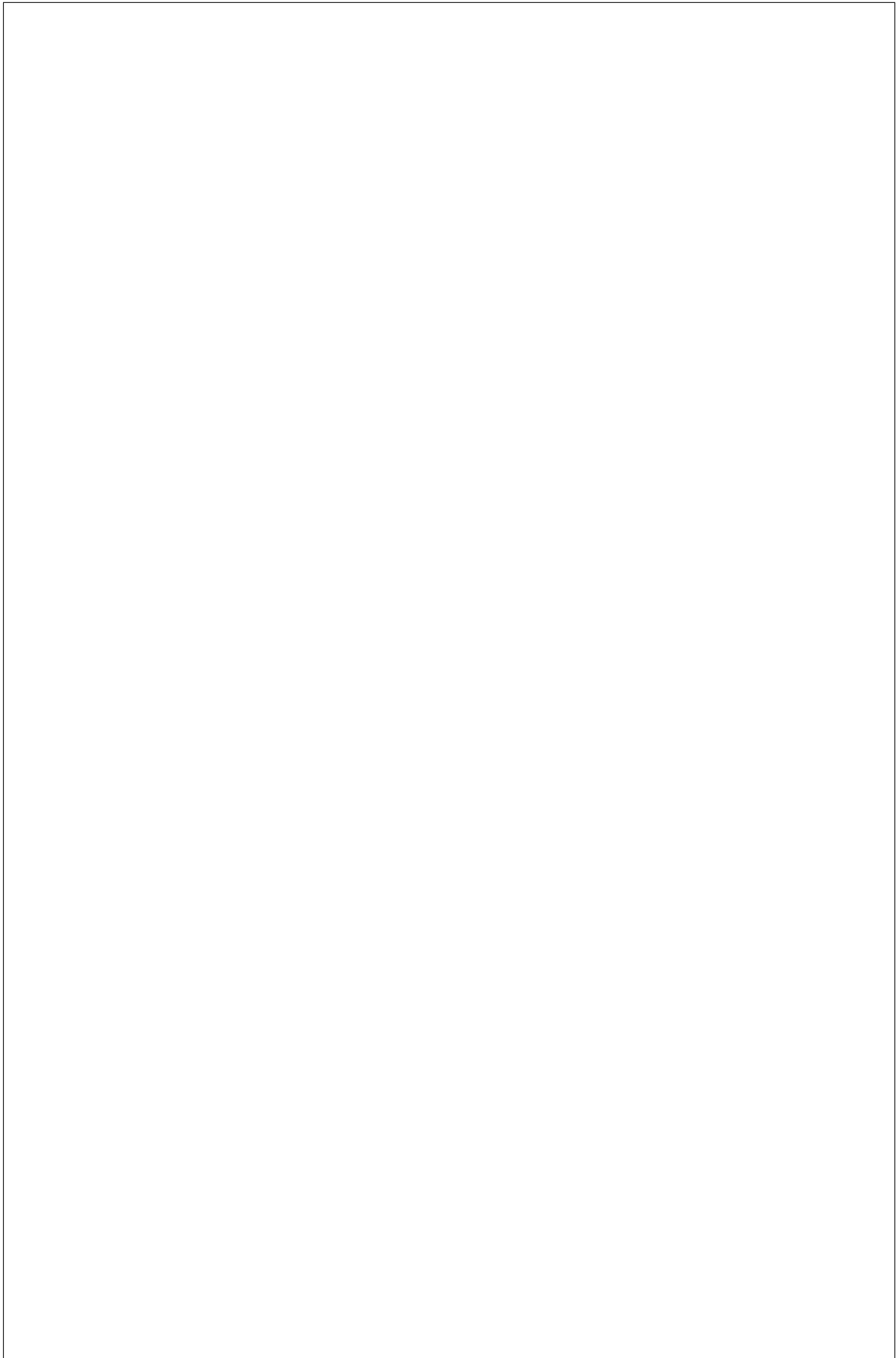




**Question 2**

Let  $X = -X^T \in \mathbb{R}^{N \times N}$  be a real  $N \times N$  antisymmetric matrix. Prove that

- (a)  $\det[X] = 0$  whenever  $N$  is odd and show in this case that 0 is an eigenvalue of  $X$ ;
- (b) all eigenvalues are imaginary and come in complex conjugate pairs, meaning when  $\lambda$  is an eigenvalue then the complex conjugate  $\lambda^* = -\lambda$  is also an eigenvalue.
- (c) if  $v \in \mathbb{C}^N$  is an eigenvector of  $X$  to the eigenvalue  $\lambda$ , then  $v^*$  is an eigenvector of  $X$  to the eigenvalue  $\lambda^* = -\lambda$ .



**Question 3**

- (a) Let  $a, b \in \mathbb{C}$  be two fixed complex numbers and  $a$  has a positive real part  $\operatorname{Re}(a) > 0$ . Prove the following integral:

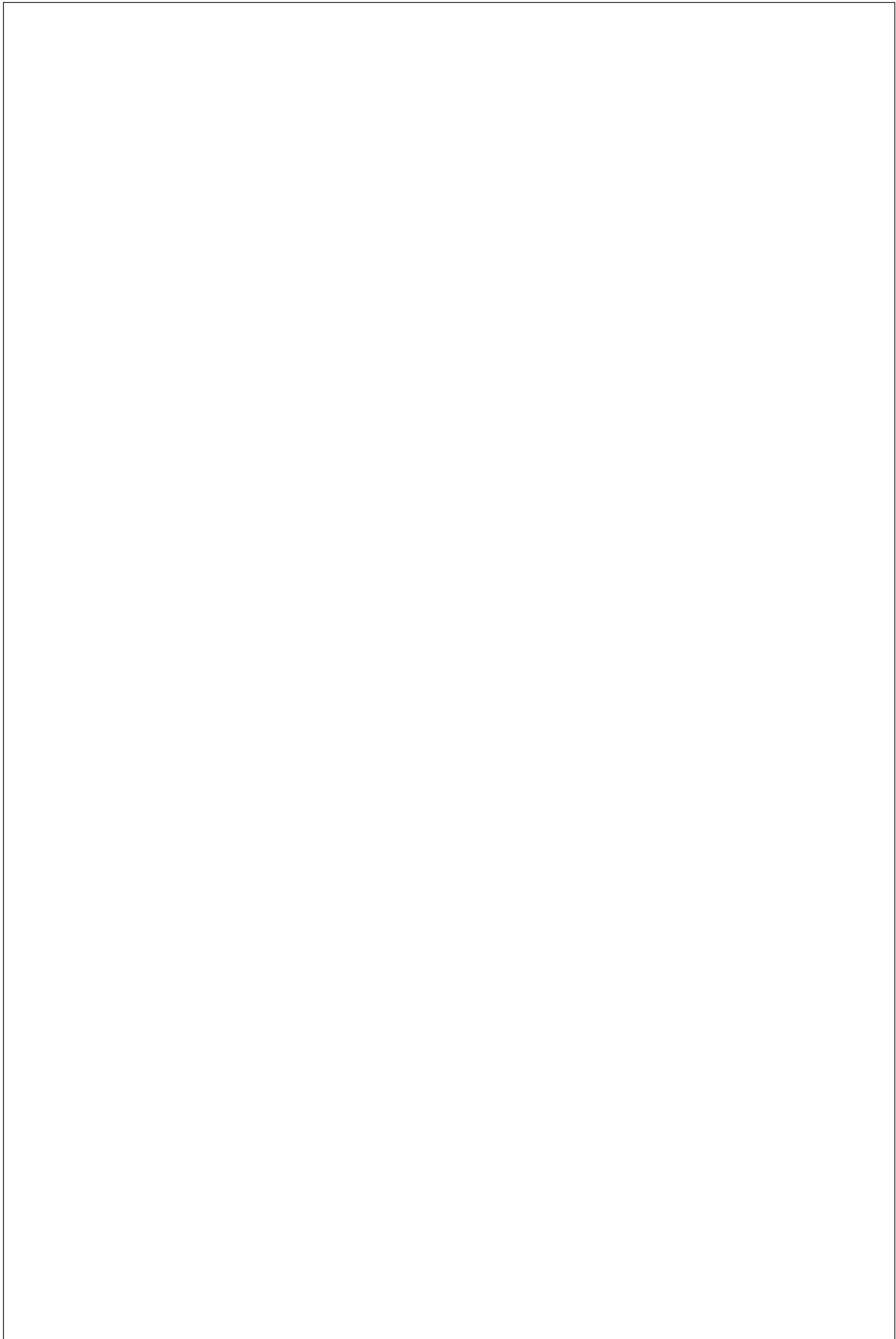
$$\int_{-\infty}^{\infty} \exp[-ax^2 + 2bx] dx = \frac{\sqrt{\pi}}{\sqrt{a}} \exp\left[\frac{b^2}{a}\right],$$

where  $\sqrt{a}$  is the principal value of the square root of  $a$  meaning it has a branch cut along the negative real line with the square root of a positive number being positive.

- (b) Let  $A \in \mathbb{R}^{3 \times 3}$  be an invertible  $3 \times 3$  real matrix. Compute the Gaussian integral

$$I(A) = \int_{\mathbb{R}^3} \exp[-x^T A x] d^3 x$$

where  $x = (x_1, x_2, x_3)^T \in \mathbb{R}^3$  is a three dimensional column vector and the volume element is  $d^3 x = dx_1 dx_2 dx_3$ .





**End of Assignment**